The ZX calculus is incomplete for quantum mechanics

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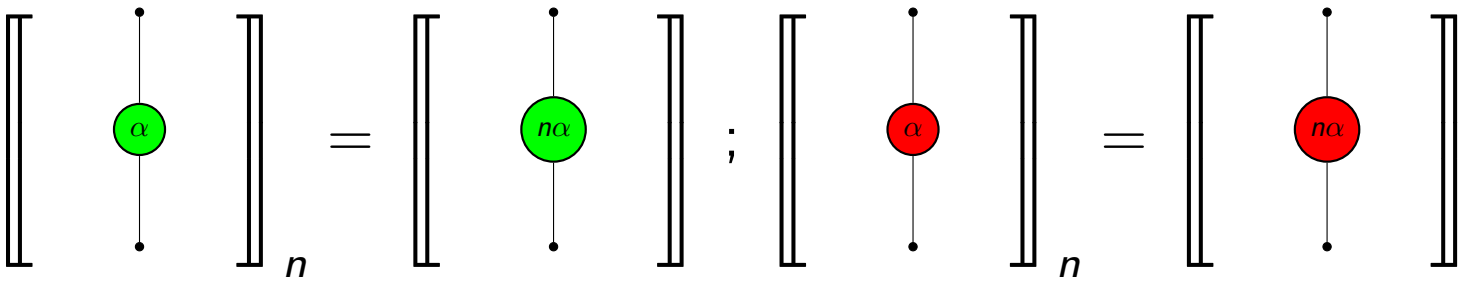
Attribution

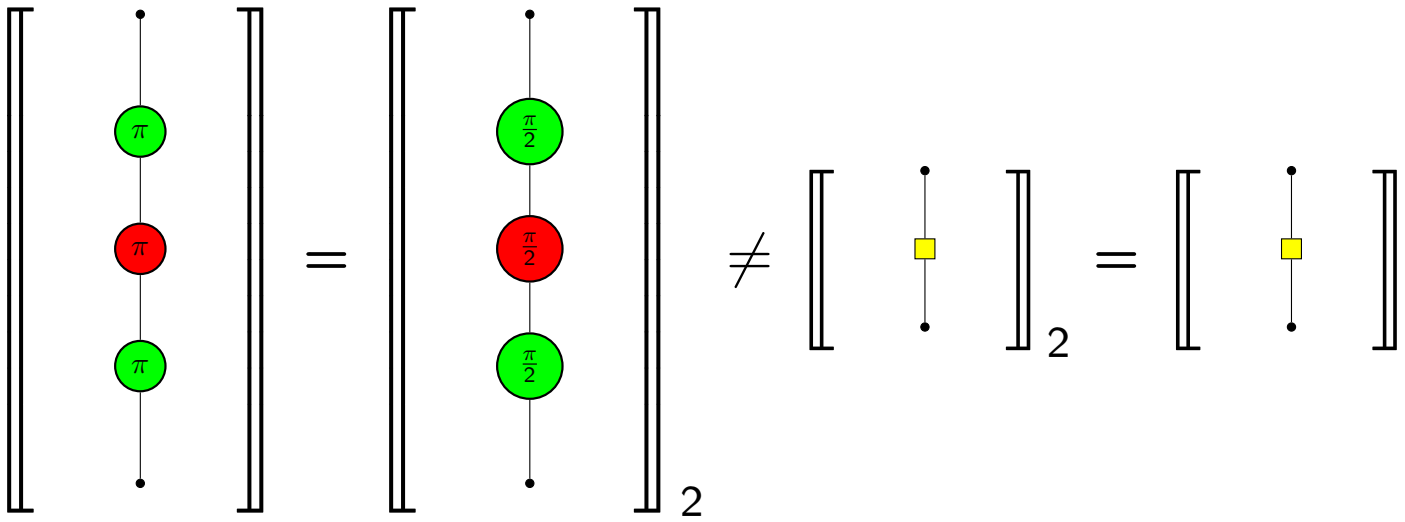
- I provided the initial motivating counter example
- Christian Schroeder finished the proof by using Duncan and Perdrix's argument
- Christian uses a different example in his master thesis

Alternative Model

- Ross Duncan and Simon Perdrix prove that the Euler decomposition of the Hadamard gate is not derivable from the rules of the ZX calculus in [1].
- They do so by defining a new class of models for the ZX calculus
- The new interpretations multiply the angles in the diagram with an integer
- The ZX calculus preserves soundness for these models

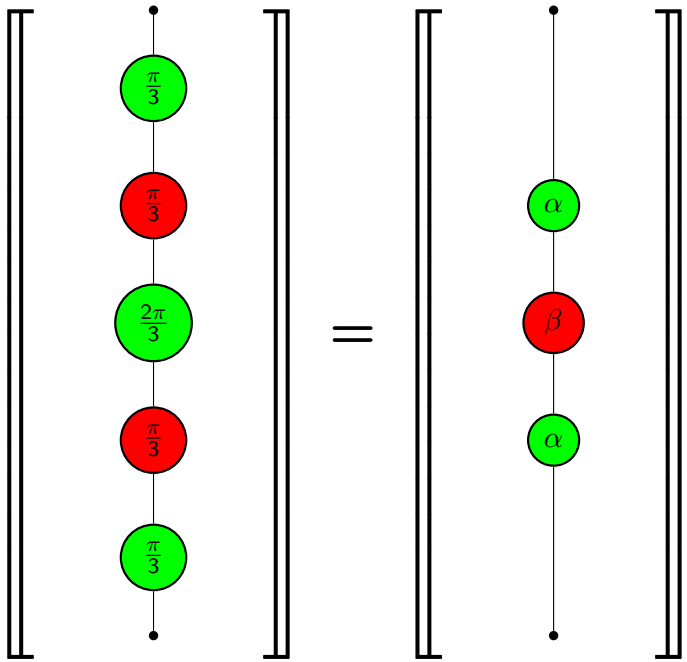


Necessity of Euler decomposition



Therefore, the ZX calculus (without Euler decomposition) is incomplete.

Standard interpretations are equal



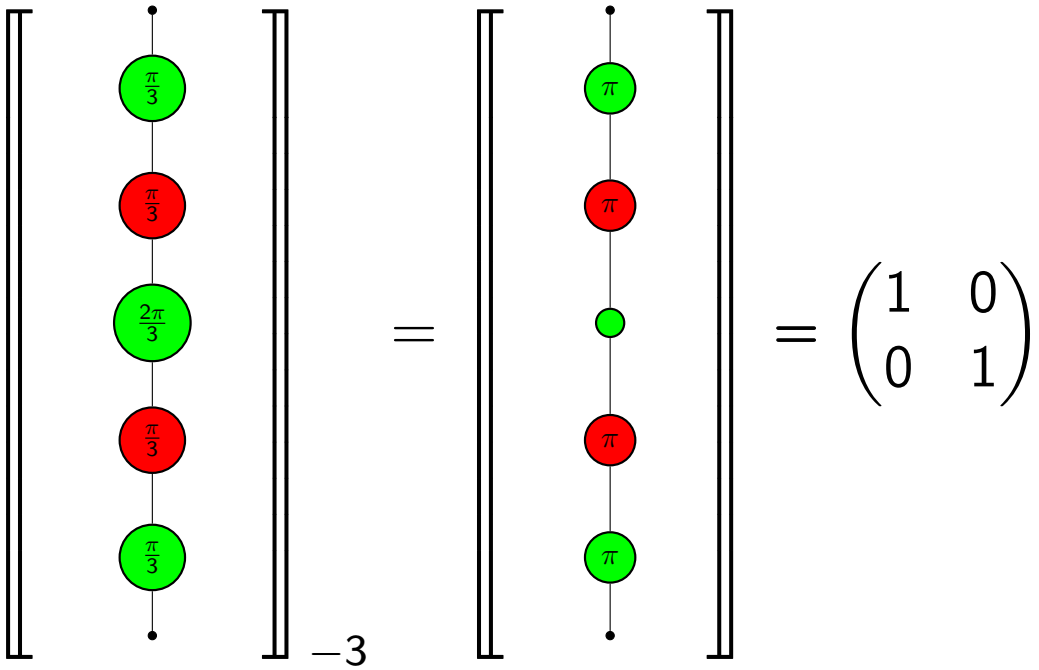
$$\alpha := -\arccos\left(\frac{5}{2\sqrt{13}}\right) \approx 0.2561\pi$$

$$\beta := -2\arcsin\left(\frac{\sqrt{3}}{4}\right) \approx -0.2851\pi$$

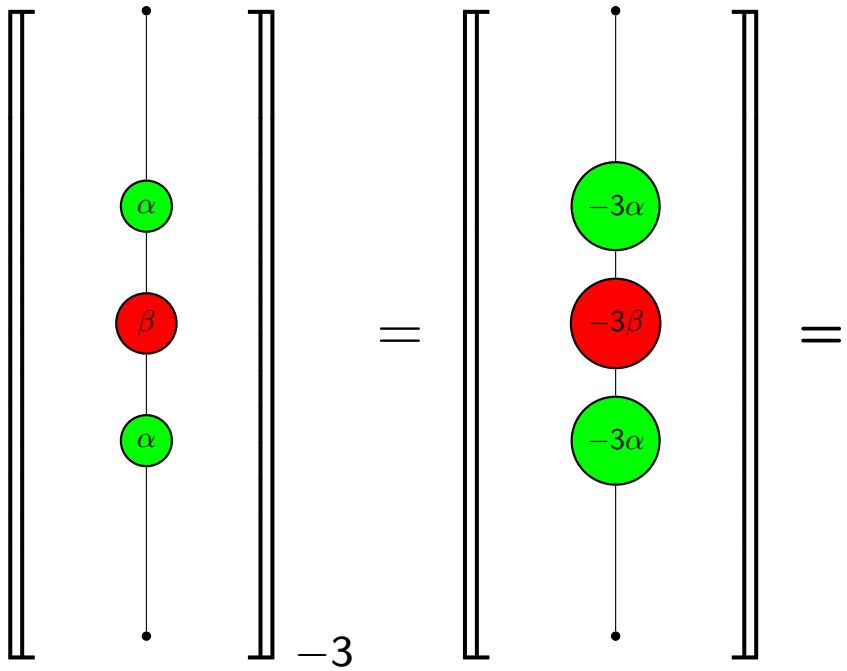
$$\phi := \arcsin\left(\frac{\sqrt{3}}{4}\right) - \alpha \approx 0.3987\pi$$

$e^{i\phi}$ is the scalar up to which they are equal

Alternative intepretations are not equal



Alternative interpretations are not equal



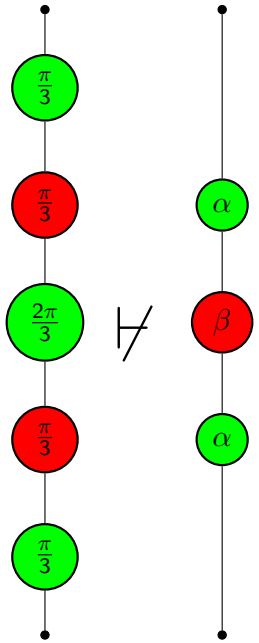
$$\begin{pmatrix} 0.0508 + 0.2196i & -0.5628 + 0.7953i \\ -0.5628 + 0.7953i & 0.2239 - 0.0252i \end{pmatrix}$$

Incomplete

However, both interpretations $[\![\cdot]\!]$ and $[\![\cdot]\!]_{-3}$ are sound. This means,

$$D_1 \vdash D_2 \quad \Rightarrow \quad [\![D_1]\!] = [\![D_2]\!] \quad \wedge \quad [\![D_1]\!]_{-3} = [\![D_2]\!]_{-3}$$

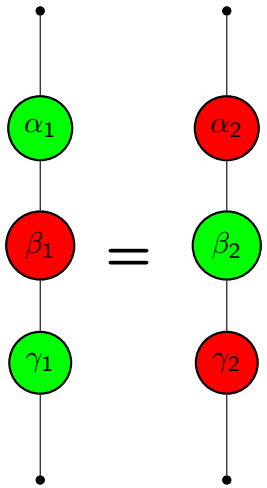
Therefore,



and the ZX calculus is incomplete for quantum mechanics.

Conclusion and future work

- ZX complete for stabilizer quantum mechanics, so not that bad
- Need to extend calculus by introducing new rules
- A color swap rule might be a good idea



- Closed form solution for $\alpha_2, \beta_2, \gamma_2$?


R. Duncan and S. Perdrix.
Graph States and the Necessity of Euler Decomposition.
In *CiE '09 Proceedings of the 5th Conference on Computability in Europe: Mathematical Theory and Computational Practice* , pages 167–177. Springer, 2009.